UNIVERSALJOURNAL

ISSN 2721-7132

CALCULUS – DIFFERENTIAL, INTEGRAL, AND INFINITE

Garry Alfonsius

Bachelor of Science in Business Administration at Universal Institute of Professional Management - UIPM Singapore

A. ABSTRACT

Calculus, originally called infinitesimal calculus or "the calculus of infinitesimals", is the mathematical study of continuous change, in the same way that geometry is the study of shape and algebra is the study of generalizations of arithmetic operations.

It has two major branches, differential calculus and integral calculus; the former concerns instantaneous rates of change, and the slopes of curves, while integral calculus concerns accumulation of quantities, and areas under or between curves. These two branches are related to each other by the fundamental theorem of calculus, and they make use of the fundamental notions of convergence of infinite sequences and infinite series to a well-defined limit.

Keywords: Differential, Integral, and infinite

B. INTRODUCTION

The infinitely small and the infinitely large – in one form or another – are essential in calculus. In fact, they are among the distinguishing features of calculus compared to many other branches of mathematics (for example algebra). They have appeared throughout the history of calculus in various guises: infinitesimals, indivisibles, differentials, evanescent quantities, moments, infinitely large and infinitely small magnitudes, infinite sums, power series, limits, and hyperreal numbers. And they have been fundamental at both the technical and conceptual levels – as underlying tools of the subject and as its foundational underpinnings. We will consider examples of these aspects of the infinitely small and large as they unfolded in the history of calculus from the 17th through the 20th centuries.

C. METHOD

1. Differential

Differential calculus deals with the rate of change of one quantity with respect to another. Or you can consider it as a study of rates of change of quantities. For example, velocity is the rate of change of distance with respect to time in a particular direction. If f(x) is a function, then f'(x) = dy/dx is the differential equation, where f'(x) is the derivative of the function, y is dependent variable and x is an independent variable.

For example, in physics, the derivative of the displacement of a moving body with respect to time is the velocity of the body, and the derivative of velocity with respect to time is acceleration. The derivative of the momentum of a body with respect
to time equals the force applied to the body; rearranging this derivative statement leads to the famous **F = *m*a** equation associated with Newton's second law of motion. The reaction rate of a chemical reaction is a derivative. In operations research, derivatives determine the most efficient ways to transport materials and design factories.

In differential calculus basics, you may have learned about differential equations, derivatives, and applications of derivatives. For any given value, the derivative of the function is defined as the rate of change of functions with respect to the given values. Differentiation is a process where we find the derivative of a function. Let us discuss the important terms involved in the differential calculus basics.

a) **Functions**

A function is defined as a relation from a set of inputs to the set of outputs in which each input is exactly associated with one output. The function is represented by "f(x)".

b) **Dependent Variable**

The dependent variable is a variable whose value always depends and determined by using the other variable called an independent variable. The dependent variable is also called the outcome variable. The result is being evaluated from the mathematical expression using an independent variable is called a dependent variable.

c) **Independent Variable**

Independent variables are the inputs to the functions that define the quantity which is being manipulated in an experiment. Let us consider an example y= 3x. Here, x is known as the independent variable and y is known as the dependent variable as the value of y is completely dependent on the value of x.

d) **Domain and Range**

The domain of a function is simply defined as the input values of a function

and range is defined as the output value of a function. Take an example, if $f(x) = 3x$
be a function, the domain values or the input values are {1, 2, 3} then the range of a function is given as

$f(1) = 3(1) = 3$

$f(2) = 3(2) = 6$

$f(3) = 3(3) = 9$

Therefore, the range of the function will be {3, 6, 9}.

e) **Limits**

The limit is an important thing in calculus. Limits are used to define the continuity, integrals, and derivatives in the calculus. The limit of a function is defined as follows:

Let us take the function as “f” which is defined on some open interval that contains some numbers, say “a”, except possibly at “a” itself, then the limit of a function f(x) is written as:

limx→af(x)=L, *iff* given e > 0, there exists d > 0 such that $0 < |x – a| < d$ implies that $|f(x) – L| < e$
It means that the limit f(x) as “x” approaches “a” is “L”

f) **Interval**

An interval is defined as the range of numbers that are present between the two given numbers. intervals can be classified into two types namely:

- **Open Interva**l – The open interval is defined as the set of all real numbers x such that $a < x < b$. It is represented as (a, b)
- **Closed Interval** – The closed interval is defined as the set of all real numbers x such that $a \le x$ and $x \le b$, or more concisely, $a \le x \le b$, and it is represented by [a, b]

2. Integral

The first documented systematic technique capable of determining integrals is the method of exhaustion of the ancient Greek astronomer Eudoxus (*ca.* 370 BC),
which sought to find areas and volumes by breaking them up into an infinite number of divisions for which the area or volume was known. This method was further developed and employed by Archimedes in the 3rd century BC and used to calculate the area of a circle, the surface area and volume of a sphere, area of an ellipse, the area under a parabola, the volume of a segment of a paraboloid of revolution, the volume of a segment of a hyperboloid of revolution, and the area of a spiral.

A similar method was independently developed in China around the 3rd century AD by Liu Hui, who used it to find the area of the circle. This method was later used in the 5th century by Chinese father-and-son mathematicians Zu Chongzhi and Zu Geng to find the volume of a sphere.

In the Middle East, Hasan Ibn al-Haytham, Latinized as Alhazen (c. 965 – c. 1040 AD) derived a formula for the sum of fourth powers. He used the results to carry out what would now be called an integration of this function, where the formulae for the sums of integral squares and fourth powers allowed him to calculate the volume of a paraboloid.

The next significant advances in integral calculus did not begin to appear until the 17th century. At this time, the work of Cavalieri with his method of Indivisibles, and work by Fermat, began to lay the foundations of modern calculus, with Cavalieri computing the integrals of x^n up to degree $n = 9$ in Cavalieri's quadrature formula. Further steps were made in the early 17th century by Barrow and Torricelli, who provided the first hints of a connection between integration and differentiation.
Barrow provided the first proof of the fundamental theorem of calculus. Wallis generalized Cavalieri's method, computing integrals of x to a general power, including negative powers and fractional powers.

The branch of mathematics in which the notion of an integral, its properties and methods of calculation are studied. Integral calculus is intimately related to differential calculus, and together with it constitutes the foundation of mathematical analysis.

The indefinite integral of a given real-valued function on an interval on the real axis is defined as the collection of all its primitives on that interval, that is, functions whose derivatives are the given function. The indefinite integral of a function ff is denoted by ∫f(x)dx∫f(x)dx. If FF is some primitive of ff, then any other primitive of it has the form F+CF+C, where CC is an arbitrary constant; one therefore writes
∫f(x)dx=F(x)+C. The operation of finding an indefinite integral is called integration. Integration is the operation inverse to that of differentiation:

$$\int F'(x)dx=F(x)+C, \ d\int f(x)dx=f(x)dx$$

The operation of integration is linear: If on some interval the indefinite integrals

∫f1(x)dx and ∫f2(x)dx∫f1(x)dx and ∫f2(x)dx

exist, then for any real numbers λ1λ1 and λ2λ2, the following integral exists on this interval:

∫[λ1f1(x)+λ2f2(x)] dx∫[λ1f1(x)+λ2f2(x)] dx

and equals

$\lambda 1\int f1(x)dx+\lambda 2\int f2(x)dx.\lambda 1\int f1(x)dx+\lambda 2\int f2(x)dx.$

For indefinite integrals, the formula of integration by parts holds: If two functions uu and vv are differentiable on some interval and if the integral $\int vdu\int vdu$ exists, then so does the integral $\int udv\int udv$, and the following formula holds:

$$\int udv=uv-\int vdu.\int udv=uv-\int vdu.$$

The formula for change of variables holds: If for two functions ff and $\varphi\varphi$ defined on certain intervals, the composite function $f\circ\varphi f\circ\varphi$ makes sense and the function $\varphi\varphi$ is differentiable, then the integral

$$\int f[\varphi(t)]\varphi'(t)dt\int f[\varphi(t)]\varphi'(t)dt$$

exists and equals (see Integration by substitution)

$$\int f(x)dx.$$

3. Infinite

Infinite Calculus covers all of the fundamentals of Calculus: limits, continuity, differentiation, and integration as well as applications such as related rates and finding volume using the cylindrical shell method. Designed for all levels of learners, from beginning to advanced. A term which formerly included various branches of mathematical analysis connected with the concept of an infinitely-small function.

Even though the method of "infinitely smalls" had been successfully employed in various forms by the scientists of Ancient Greece and of Europe in the Middle Ages to solve problems in geometry and in natural science, exact definitions of the fundamental concepts of the theory of infinitely-small functions were laid only in the 19th century. In order to grasp the importance of this method, it must be pointed out that it was not the infinitesimal calculus itself which was of practical importance, but only the cases in which its use resulted in finite quantities. Three kinds of such problems were particularly important in the history of mathematics.

1) The simplest problems, solved by the mathematicians of Ancient Greece by the method of exhaustion (cf. Exhaustion, method of), in which infinitesimal quantities are used merely to prove that two given magnitudes (or two ratios between given magnitudes) are equal.

2) More sophisticated problems involving the method of exhaustion, in which the required finite magnitude is obtained as the limit of a sum

$$\Delta(1n)+\cdots+\Delta(nn)\ (n\to\infty)\Delta 1(n)+\cdots+\Delta n(n)\ (n\to\infty)$$

of an infinitely-large number of infinitely-small quantities. These problems ultimately gave rise to integral calculus.

3) Problems in which the finite magnitude is obtained as the limit of ratios of infinitely-small magnitudes; they gave rise to differential calculus.

The invention of the method of exhaustion is attributed to Eudoxus of Cnidos (4th century B.C.). However, this may be, the method is used throughout Book 12 of Euclid's Elements as the principal deductive tool. Euclid's chain of reasoning may be written in modern form as follows: If all the ratios

$$a1/b1=\cdots=$$

$$an/bn=\cdots= k$$

are equal to each other and to a constant value kk, and if, as n→∞n→∞, both differences a−an, b−bn become infinitely small, then

$$a/b=k.$$

D. CONCLUSION

We already know about differential, integral, and infinite. Hopefully you will understand it and make the math or calculus the easy way.

E. REFERENCES

1. DeBaggis, Henry F.; Miller, Kenneth S. (1966). *Foundations of the Calculus*. Philadelphia: Saunders. OCLC 527896
2. "Differential Calculus" byjus.com. available at: https://byjus.com/maths/differential- calculus
3. *Heath, Thomas Little (1897). The Works of Archimedes. England: Cambridge University Publications.*
4. "Integral Calculus" encyclopediaofmath.org. available at: https://encyclopediaofmath.org/wiki/Integral_calculus
5. "Infinitesimal Calculus" encyclopediaofmath.org. available at: https://encyclopediaofmath.org/wiki/Infinitesimal_calculus
6. Katz, Victor J. (June 1995). "Ideas of Calculus in Islam and India". *Mathematics Magazine*. 68 (3): 163–174. doi:10.2307/2691411
7. Kleiner, I. (2001). History of the infinitely small and the infinitely large in calculus. *Educational Studies in Mathematics*, *48*(2-3), 137-174.
8. *Struik, Dirk J., ed. (1986). A Source Book in Mathematics, 1200–1800. Princeton, NJ: Princeton University Press. p. 214. ISBN 0-691-08404-1*

9 772721 713200